

On the origin of the stellar halo and multiple stellar populations in the globular cluster NGC 1851

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ABSTRACT

We propose that the observed stellar halo around the globular cluster (GC) NGC 1851 is evidence for its formation in the central region of its defunct host dwarf galaxy. We numerically investigate the long-term dynamical evolution of a nucleated dwarf galaxy embedded in a massive dark matter halo under the strong tidal field of the Galaxy. The dwarf galaxy is assumed to have a stellar nucleus (or a nuclear star cluster) that could be the progenitor for NGC 1851. We find that although the dark matter halo and the stellar envelope of the host dwarf of NGC 1851 can be almost completely stripped during its orbital evolution around the Galaxy, a minor fraction of stars in the dwarf can remain trapped by the gravitational field of the nucleus. The stripped nucleus can be observed as NGC 1851 with no/little dark matter whereas stars around the nucleus can be observed as a diffuse stellar halo around NGC 1851. The simulated stellar halo has a symmetric distribution with a power-law density slope of ~ -2 and shows no tidal tails within ~ 200 pc from NGC 1851. We show that two GCs can merge with each other to form a new nuclear GC embedded in field stars owing to the low stellar velocity dispersion of the host dwarf. This result makes no assumption on the ages and/or chemical abundances of the two merging GCs. Thus the observed stellar halo and characteristic multiple stellar populations in NGC 1851 suggest that NGC 1851 could have formed initially in the central region of an ancient dwarf galaxy. We predict that the stellar halo of NGC 1851 may have at least three different stellar populations. We also suggest some Galactic GCs with diffuse halos, such as NGC 1904 and NGC 5694, could be formed in a similar way as NGC 1851. We discuss the importance of GC merging within dwarfs in the formation of multiple stellar populations with abundance spreads in heavy elements in some Galactic GCs, such as M22 and NGC 2419. We also discuss other possible scenarios for the formation of the stellar halo around NGC 1851.

Key words: globular cluster: general – galaxies: star clusters: general – galaxies: stellar content – stars: formation

1 INTRODUCTION

The Milky Way galaxy hosts about 150 globular clusters (hereafter GCs) (Harris 1996, – 2010 edition) and these objects have been regarded by many as the best examples of a simple stellar population, i.e., single age, helium abundance, metallicity, and initial mass function (Renzini, & Buzzoni 1986). However, many studies have revealed that these simple stellar populations display extremely large chemical abundance variations for the light elements (e.g., see reviews by Smith 1987; Kraft 1994; Gratton, Sneden, &

Carretta 2004). Although the exact origin(s) of these light element variations remains unclear, it has become clear that every well studied GC shows light element abundance variations.

The massive GC ω Cen possesses a large spread in light element, iron-peak element, and neutron-capture element abundances (e.g., Freeman & Rodgers 1975; Cohen 1981; Norris & Da Costa 1995; Smith et al. 2000), in contrast to the vast majority of GCs, and as such, ω Cen is regarded as an exception. Indeed, it has been strongly argued that ω Cen is the remnant nucleus of an accreted dwarf galaxy (e.g., Freeman 1993). Photometric (Bedin et al. 2004) and spec-

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troscopic (Johnson & Pilachowski 2010) studies now demonstrate the true complexity of this system.

Additionally, photometric and spectroscopic studies of other GCs have revealed complex structure in color-magnitude diagrams (M22: Piotto 2008; NGC 1851: Milone et al. 2008; 47 Tuc: Anderson et al. 2009) as well as perhaps unexpected dispersions in heavy element abundances (M22: Marino et al. 2009; NGC 1851: Yong & Grundahl 2008; Carretta et al. 2010, 2011; Villanova et al. 2010). At present, there are a number of GCs which clearly display multiple populations. Of these clusters, NGC 1851 may hold particular importance. NGC 1851 has a bi-modal horizontal branch, a double subgiant branch, a large dispersion in *s*-process elements abundances, and a dispersion in metallicity (Yong & Grundahl 2008; Carretta et al. 2010, 2011; Villanova et al. 2010). Milone et al. (2011) have recently shown that two stellar populations with different *s*-element abundances are associated with two distinct populations in sub-giant and red-giant branches in NGC 1851. An intriguing, but tentative, speculation is that NGC 1851 may be the product of the merger of two individual clusters (Carretta et al. 2010, 2011). Although recent observations have revealed two distinct populations in the horizontal branch stars (Salaris et al. 2008) and in red giant branch ones (Han et al. 2009) and a possible bimodal [Fe/H] distribution (Lee et al. 2009) within NGC 1851, it remains unclear how these observations can be explained in the context of GC merging.

Recently Olszewski et al. (2010) revealed that NGC 1851 is surrounded by a diffuse stellar halo with a size of more than 500 pc and a mass of about 0.1% of the dynamical mass of NGC 1851. The halo has an symmetric distribution with no elongated tidal tails and a power-law radial density profile that can fit to $r^{-1.24}$. Diffuse stellar halos surrounding GCs have been discovered also in Whiting 1 (Carraro et al. 2007), AM 4 (Carraro et al. 2009), NGC 5694 (Correnti et al. 2011), and NGC 1904 (Carballo-Bello & Martínez-Delgado 2011), which implies that stellar halos around GCs are not very rare objects. Olszewski et al. (2009) suggested that the observed diffuse stellar halo can be formed from destruction of a dwarf galaxy that previously contained NGC 1851. Given the presence of stellar streams associated with GCs in the Galaxy and M31 (e.g. Bellazzini et al. 2003; Mackey et al. 2010), the above suggestion seems to be quite reasonable.

Our previous numerical studies showed that nucleated dwarfs can be transformed into very massive GCs such as ω Cen in the Galaxy (Bekki & Freeman 2003) and G1 in M31 (Bekki & Chiba 2004) or ultra-compact dwarf galaxies (UCDs) in clusters of galaxies (Bekki et al. 2003). These studies demonstrated that if the orbits of nucleated dwarfs are highly radial, then strong tidal fields of galaxies and clusters of galaxies can strip almost all of the dark matter and field stars from nucleated dwarfs while the nuclei remain intact owing to their compactness. They thus claimed that the stripped nuclei of nucleated dwarfs can be observed as massive GCs or UCDs. Bekki et al. (2003) already showed that the stripped nuclei can be surrounded by diffuse stellar halos that are composed of field stars left behind from the tidal destruction processes of the dwarfs (see their Figs. 2 and 3). However they did not investigate the physical properties of the halos around stripped nuclei and their models for the formation of UCDs are unreasonable for the forma-

tion of NGC 1851, a system much less massive than UCDs. Numerical simulations on dwarf destruction process (e.g., Goerdt et al. 2008; Lokas et al. 2010) and nucleus formation from GC merging (e.g., Capuzzo-Dolcetta & Miocchi 2008a, b) did not discuss the origin of NGC 1851 either. Thus it is unclear whether the observed stellar halo around NGC 1851 can be really formed from disruption of a nucleated dwarf interacting with the Galaxy.

The purpose of this paper is to explore whether the stellar halo around NGC 1851 can be formed as a result of transformation from a nucleated dwarf into a GC by using numerical simulations on the long-term dynamical evolution of nucleated dwarfs around the Galaxy. We also investigate whether GC merging, which is suggested to be responsible for the formation of NGC 1851 (e.g., Carretta et al. 2010), can really occur in the central region of the host dwarf. In the present paper, we adopt a scenario in which NGC 1851 was previously a stellar nucleus (or nuclear star cluster) that was formed in situ or through merging of two GCs with different chemical abundances. We mainly discuss (i) physical conditions for a nucleated dwarf to become NGC 1851 and (ii) time scales of GC merging in the NGC 1851's host. We also discuss how GC merging is important for the formation of GCs with abundance spreads in heavy elements, such as NGC 2419 and M22.

It should be stressed here that the present scenario is only one of possible ones. For example, Küpper et al. (2010) have recently showed that extra-tidal stellar populations can be formed as a result of tidal interaction of star clusters with their parent host galaxies. Although the present models are idealized and less realistic in some points (e.g., fixed Galactic potential and point-mass representation of GCs in dwarfs), we consider that the models enable us to grasp some essential ingredients of the formation of NGC 1851. Furthermore, none of the results presented make any assumption on the ages and/or chemical compositions of the merging GCs. This study represents an important first step towards a fully self-consistent model for the formation of NGC 1851: a considerably more sophisticated theoretical model for both dynamical and chemical properties of NGC 1851 will be constructed in our future papers.

The plan of the paper is as follows: in the next section, we describe our numerical model for dynamical evolution of nucleated dwarfs around the Galaxy and for GC merging in dwarfs. In §3, we present the numerical results on the transformation from nucleated dwarfs into GCs during disruption of dwarfs by the Galaxy. In this section, we also discuss the time scales of GC merging for different models. In §4, we provide important implications of the present results in terms of the origin of multiple stellar populations in GCs. We summarize our conclusions in §5. Note that we do not intend to discuss chemical properties of multiple stellar populations in NGC 1851 using some theoretical models: Ventura et al. (2009) recently have discussed C+N+O abundances of NGC 1851 using their models for AGB stars.

2 THE MODEL

2.1 A scenario

We consider that a nucleated dwarf galaxy is a host for NGC 1851 and was accreted onto the Galaxy at least several Gyr

Table 1. Description of the model parameters for the representative models.

Model no. ^a	$M_{s,dw}$ ^b	$R_{s,dw}$ ^c	f_{dm} ^d	f_n ^e	R_i ^f	f_v ^g	m_{gc} ^h	r_1 ⁱ	r_2 ^j	comments
M1	1.0	1.8	9	10^{-2}	35.0	0.4	-	-	-	standard model
M2	1.0	1.8	9	10^{-2}	35.0	0.2	-	-	-	more radial orbit
M3	1.0	1.8	9	10^{-2}	35.0	0.6	-	-	-	
M4	1.0	1.8	9	10^{-2}	35.0	0.8	-	-	-	
M5	1.0	1.8	9	10^{-2}	8.8	0.4	-	-	-	smaller apocenter
M6	1.0	1.8	9	10^{-2}	17.5	0.4	-	-	-	
M7	1.0	1.8	9	10^{-2}	70.0	0.4	-	-	-	
M8	0.1	0.6	9	10^{-2}	35.0	0.4	-	-	-	smaller dwarf mass
M9	0.5	1.3	9	10^{-2}	35.0	0.4	-	-	-	
M10	2.0	2.5	9	10^{-2}	35.0	0.4	-	-	-	
M11	1.0	1.8	9	10^{-4}	35.0	0.4	-	-	-	less massive nucleus
M12	1.0	1.8	9	10^{-3}	35.0	0.4	-	-	-	
M13	1.0	1.8	9	10^{-1}	35.0	0.4	-	-	-	
M14	0.1	2.8	90	5×10^{-2}	35.0	0.4	-	-	-	LSB
M15	0.1	2.8	90	5×10^{-2}	35.0	0.2	-	-	-	
M16	0.1	0.6	9	5×10^{-2}	35.0	0.4	-	-	-	
M17	0.1	0.6	9	5×10^{-2}	35.0	0.2	-	-	-	
MG1	1.0	1.8	9	-	35.0	0.4	5.0	0.2	0.2	with two GCs
MG2	1.0	1.8	9	-	35.0	0.4	5.0	0.4	0.4	
MG3	1.0	1.8	9	-	35.0	0.4	5.0	0.8	0.8	
MG4	1.0	1.8	9	-	35.0	0.4	5.0	0.1	0.3	
MG5	1.0	1.8	9	-	35.0	0.4	5.0	0.1	0.5	
MG6	1.0	1.8	9	-	35.0	0.4	5.0	0.3	0.5	
MG7	1.0	1.8	9	-	35.0	0.4	0.2	0.2	0.2	
MG8	1.0	1.8	9	-	35.0	0.4	0.5	0.2	0.2	
MG9	1.0	1.8	9	-	35.0	0.4	2.0	0.2	0.2	
MGM1	1.0	1.8	9	-	35.0	0.4	5.0	0.2	0.2	GC merging
MGM2	1.0	1.8	9	-	35.0	0.4	5.0	0.1	0.4	
MGM3	1.0	1.8	9	-	35.0	0.4	5.0	0.4	0.4	
MGM4	1.0	1.8	9	-	35.0	0.4	0.2	0.2	0.2	
MGM5	1.0	1.8	9	-	35.0	0.4	0.5	0.2	0.2	
MGM6	1.0	1.8	9	-	35.0	0.4	2.0	0.2	0.2	
MGM7	0.1	2.8	90	-	35.0	0.4	5.0	0.2	0.2	LSB

^a GCs are not included in the models M1-M17 whereas they are included in MG1-9 and MGM1-7. Merging of two GCs and the resultant formation of a new single GC are included only in models MGM1-7.

^b The total mass of a stellar envelope in a dwarf in units of $10^8 M_\odot$.

^c The initial size of a stellar disk in a dwarf in units of kpc.

^d The ratio of $M_{dm,dw}$ (dark matter mass) to $M_{s,dw}$ in a dwarf.

^e The ratio of $M_{n,dw}$ (stellar nucleus mass) to $M_{s,dw}$ in a dwarf.

^f The initial distance of a dwarf from the center of the Galaxy in units of kpc.

^g The initial velocity of a dwarf is given as $f_v v_c$, where v_c is the circular velocity of the Galaxy at $R = R_i$.

^h The mass of a GC in units of $10^5 M_\odot$.

ⁱ The initial distance of GC1 from the center of its host dwarf in units of $R_{s,dw}$.

^j The initial distance of GC2 from the center of its host dwarf in units of $R_{s,dw}$.

ago. Accordingly the dwarf has been strongly influenced by the strong tidal field of the Galaxy so that it has been almost completely destroyed. The stripped nucleus (or nuclear star cluster) is now observed as one of the Galactic GCs, NGC 1851. The stellar nucleus of the NGC 1851's host dwarf was formed by merging two GCs each of which has stellar populations with almost identical chemical abundances in α and Fe-peak elements yet different ones in light elements (e.g., C, N, and O) and s -process elements. The two clusters were formed in different regions of the dwarf such that they could have slightly different abundances in s -process (and heavier) elements. After the creation of a new GC by merging of the two GCs, the new GC could quickly sink into the nuclear region owing to its larger original mass ($> 5 \times 10^5 M_\odot$). The nucleus (NGC 1851) had two distinct populations with dif-

ferent abundances in s -process elements and slightly different ages, which reflects differences in formation epochs of the two GCs.

In this scenario, it is inevitable that NGC 1851 can currently contain field stars that were initially in the nuclear region of its host. It is however unclear how many stars can finally surround NGC 1851 after the destruction of its dwarf. It is highly likely that GC merging occurs in the host dwarf owing to a small velocity dispersion of the dwarf, $\sim 10 \text{ km s}^{-1}$. However, it is unclear whether the merged GCs can become a stellar nucleus or whether the merged GCs can be stripped from the host before it can sink into the nuclear region. We thus quantitatively estimate physical properties of the stellar halo around NGC 1851 and the time scale of

GC merging and its dependences on masses and orbits of the two GCs in the present study.

2.2 Nucleated dwarf

A nucleated dwarf is modeled as a fully self-gravitating system and assumed to consist of a dark matter halo, a stellar component, and a nucleus. The dark matter halo and the main stellar component of the dwarf are represented by collisionless N-body particles. In the present N-body simulations, the nucleus is represented by a single point-mass particle, because the inner structure of NGC 1851 ($R < 10$ pc) cannot be numerically resolved. Gas dynamics and star formation including new GC formation are not included in the present study. For convenience, the stellar component (i.e., the main baryonic component) is referred to as either the “envelope” or the “stellar envelope” so that we can distinguish this component from the stellar nucleus.

The density profile of the dark matter halo with the total mass of $M_{\text{dm,dw}}$ is represented by that proposed by Salucci & Burkert (2000):

$$\rho_{\text{dm}}(r) = \frac{\rho_{\text{dm},0}}{(r + a_{\text{dm}})(r^2 + a_{\text{dm}}^2)}, \quad (1)$$

where $\rho_{\text{dm},0}$ and a_{dm} are the central dark matter density and the core (scale) radius, respectively. For convenience, we hereafter call this profile the “SB” profile (or model). Recent observational and numerical studies have shown that the adopted “cored dark matter” halos are reasonable for describing dark matter distributions in low-mass galaxies (e.g., Gavarnato et al. 2010; Oh et al. 2011). Therefore, the above SB profile rather than the “NFW” one (Navarro et al. 1996) with a central cusp predicted by the Cold Dark Matter (CDM) model is better for the present model for low-mass nucleated dwarfs. For the SB profile, the dark matter core parameters, $\rho_{\text{dm},0}$, a_{dm} , and M_0 (where M_0 is the total dark matter mass within a_{dm}) are not free parameters, and clear correlations are observed between them (Burkert 1995):

$$M_0 = 4.3 \times 10^7 \left(\frac{a_{\text{dm}}}{\text{kpc}} \right)^{7/3} M_{\odot}. \quad (2)$$

All dark matter particles are distributed within $5a_{\text{dm}}$, which can roughly correspond to the tidal radii of dwarfs (~ 7 kpc) at the apocenter distance ($=35$ kpc) for the total masses of $\sim 10^9 M_{\odot}$.

The stellar component of the dwarf is modeled as a bulge-less stellar disk with the total mass of $M_{\text{s,dw}}$ and the size of $R_{\text{s,dw}}$. The radial (R) and vertical (Z) density profiles of the stellar disk are assumed to be proportional to $\exp(-R/R_0)$ with scale length $R_0 = 0.2R_{\text{s,dw}}$ and to $\text{sech}^2(Z/Z_0)$ with scale length $Z_0 = 0.04R_{\text{s,dw}}$, respectively. In addition to the rotational velocity caused by the gravitational field of disk and dark halo components, the initial radial and azimuthal velocity dispersions are assigned to the disc component according to the epicyclic theory with Toomre’s parameter $Q = 1.5$. The vertical velocity dispersion at a given radius is set to be 0.5 times as large as the radial velocity dispersion at that point.

We investigate models with different $M_{\text{s,dw}}$ and adopt Freeman’s law (Freeman 1970) to determine R_0 of a disk galaxy according to its disk mass:

$$R_0 = C_d \left(\frac{M_d}{6 \times 10^{10} M_{\odot}} \right)^{0.5} \text{ kpc}, \quad (3)$$

where C_d is a normalization constant. Although $C_d = 3.5$ is a reasonable value for a luminous disk galaxy like the Galaxy, low-luminosity disk galaxies have low surface stellar densities and thus R_0 determined by the above equation would not be so appropriate (i.e., significantly smaller than the observed), as shown in Kauffmann et al. (2003). We thus adopt $C_d = 8.75$ for low-mass nucleated dwarfs for most models in the present study. For the adopted mass-size scaling relation, a dwarf with $M_{\text{s,dw}} = 10^8 M_{\odot}$ has $R_{\text{s,dw}} = 1.8$ kpc (stellar disk size) and $R_0 = 0.36$ kpc (scale-length). We also investigate “LSB” models in which $C = 13.8$ and therefore dwarfs are classified as very low surface brightness galaxies. These LSB models are used for discussing the origin of the observed small mass ($\sim 10^3 M_{\odot}$) of the stellar halo around NGC 1851. The ratio of $M_{\text{dm,dw}}$ to $M_{\text{s,dw}}$ is a free parameter represented by f_{dm} , though $f_{\text{dm}} = 9$ for most models.

The spin of a stellar disk of the dwarf is specified by two angles θ_d and ϕ_d (degrees). θ_d is the angle between the z -axis and the vector of the angular momentum of a disk. ϕ_d is the azimuthal angle measured from x -axis to the projection of the angular momentum vector of a disk on to the x - y plane, where the x - y plane is coincident with the Galactic plane. We have investigated models with different θ_d and ϕ_d for the orbits of the host dwarf of NGC 1851 adopted in the standard model (M1). We found that the present results on the formation processes of stellar halos around stripped nuclei are essentially the same between the models. So we describe the results of the models with $\theta_d = 45^\circ$ and $\phi_d = 30^\circ$.

The stellar nucleus with a mass $M_{\text{n,dw}}$ is represented by a point-mass particle that has a Plummer potential with a scaling length of $a_{\text{n,dw}}$. The value of $a_{\text{n,dw}}$ is set to be the same as the gravitational softening length of the stellar nucleus particle, as described later. The ratio of $M_{\text{n,dw}}$ to $M_{\text{s,dw}}$ in a dwarf is a free parameter which is represented by f_n and ranges from 10^{-4} to 10^{-1} . We investigate models with vastly different f_n , because we need to clearly demonstrate how the formation processes of the stellar halo around NGC 1851 depend on f_n . Recent observations have shown that the typical ratio of $M_{\text{n,dw}}$ to $M_{\text{s,dw}}$ in brighter nucleated galaxies ($M_B < -15$ mag) is $\sim 0.3\%$ and there is a dispersion of 3.2 in f_n (Côte et al. 2006). Possibly $M_{\text{s,dw}}$ of nucleated dwarfs at high redshifts were significantly lower than those at present, because gas had not been completely converted into stars. Therefore it is likely that f_n of ancient nucleated dwarfs that were accreted onto the Galaxy long time ago were significantly larger than 0.003. We therefore mainly investigate the models with $f_n = 0.01$. It should be noted that Georgiev et al. (2009) found bright ($M_V \sim -9$ mag) nuclear GCs in dwarf irregular galaxies, which could be progenitors of old Galactic GCs like NGC 1851.

In the present paper, we do not discuss how the destruction processes of dwarfs depend on initial dark matter profiles of the dwarfs. This is because our previous simulations (Bekki et al. 2003) have already investigated dynamical evolution of dwarfs with cored (SB) and cuspy (NFW) dark matter halos. They clearly demonstrated that if the NFW models are adopted, then the transformation from nucleated

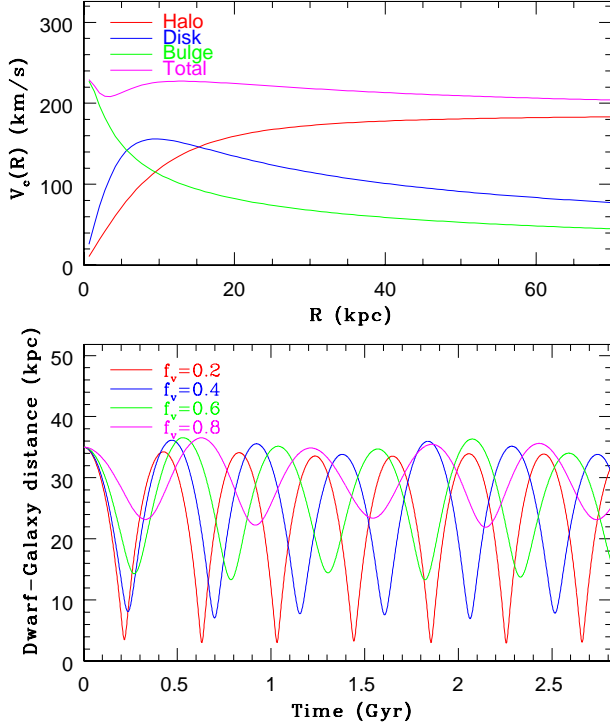


Figure 1. The rotation curve profile ($V_c(R)$) for the Galaxy model (upper) and the time evolution of the distance between a nucleated dwarf and the Galaxy for four representative models. The contributions from halo (red), disk (blue), bulge (green), and all components (magenta) are shown separately in the upper panel. The time evolution dwarf-Galaxy distances for $T < 2.8$ Gyr are shown for models with $f_v = 0.2$ (red), dwarf-Galaxy distances are shown for models with $f_v = 0.2$ (red), 0.4 (blue), 0.6 (green), and 0.8 (magenta) in the lower panel.

dwarfs into UCDs or massive GCs become much less likely. We thus consider that this previous result can be true for the formation of NGC 1851 from a nucleated dwarf.

2.3 The Galaxy

The orbit of the dwarf is assumed to be influenced only by the *fixed* gravitational potential of the Galaxy having three components: a dark matter halo, a disk, and a bulge. We assume the following logarithmic dark matter halo potential for the Galaxy,

$$\Phi_{\text{halo}} = v_{\text{halo}}^2 \ln(r^2 + d^2), \quad (4)$$

where $d = 12$ kpc, $v_{\text{halo}} = 131.5 \text{ km s}^{-1}$ and r is the distance from the center of the Galaxy. The gravitational potential of the Galactic disk is represented by a Miyamoto-Nagai (1975) potential;

$$\Phi_{\text{disk}} = -\frac{GM_{\text{disk}}}{\sqrt{R^2 + (a + \sqrt{z^2 + b^2})^2}}, \quad (5)$$

where $M_{\text{disk}} = 1.0 \times 10^{11} M_{\odot}$, and $a = 6.5$ kpc, $b = 0.26$ kpc, and $R = \sqrt{x^2 + y^2}$. The following spherical Hernquist (1990) model is adopted for the potential of the Galactic bulge;

$$\Phi_{\text{bulge}} = -\frac{GM_{\text{bulge}}}{r + c}, \quad (6)$$

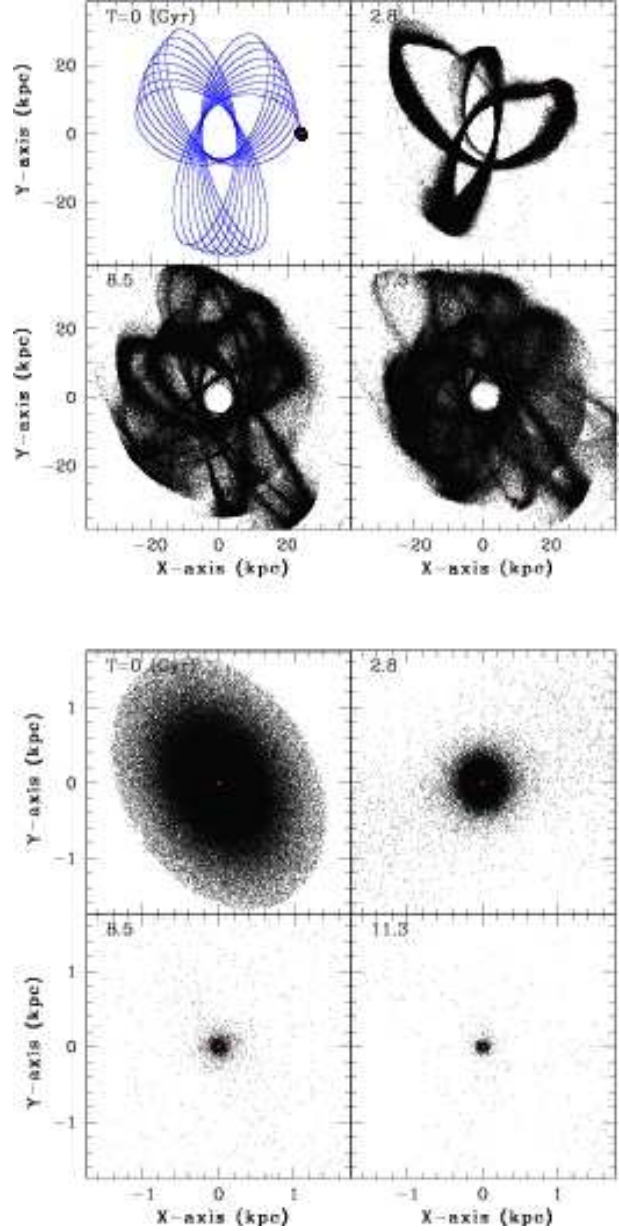


Figure 2. The time evolution of the stellar distribution of a nucleated dwarf projected onto the x - y plane for a large scale (upper) and small scale (lower) in the standard model (M1). The center of each frame is coincident with the Galactic center in the upper four and with the center of the dwarf in the lower. The location of a nucleus particle is shown by a red big dot. The blue line in the upper left panel of the upper four describes the orbital evolution of the dwarf. The time T shown in the upper left corner of each panel represents the elapsed time for this simulation.

where $M_{\text{bulge}} = 3.4 \times 10^{10} M_{\odot}$, and $c = 0.7$ kpc. This reasonable set of parameters gives a realistic rotation curve for the Galaxy with a maximum rotation speed of 224 km s^{-1} at $R = 8.5$ kpc. Fig. 1 shows the rotation curve profile ($V_c(R)$) for the adopted model.

The center of the Galaxy is always set to be $(x, y, z) = (0, 0, 0)$ whereas the initial location and velocity of the dwarf are free parameters that can control the orbital evolution of the dwarf. The initial distance of the dwarf from

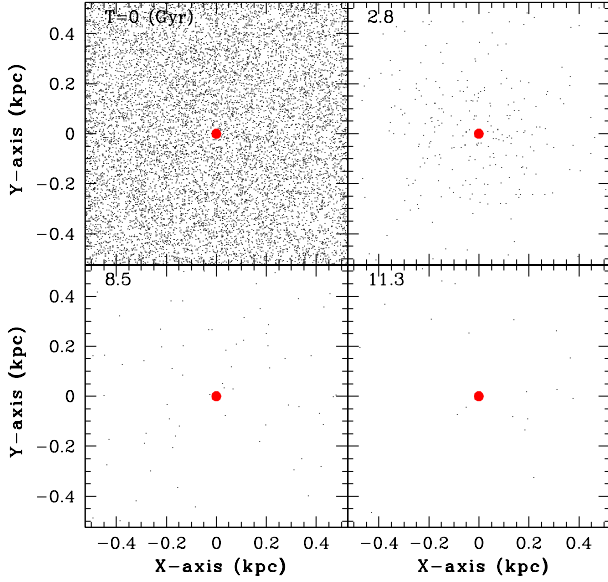


Figure 3. The distribution of dark matter particles of the nucleated dwarf projected onto the x - y plane at four representative time steps for the standard model. The central big red particle represents the location of the stellar nucleus of the dwarf.

the Galactic center and the velocity are represented by R_i and $f_v v_c$, respectively, where v_c is the circular velocity at R_i . The inclination angle between the initial orbital plane and the x - z plane (=Galactic disk plane) is denoted as θ . Guided by previous studies of orbital evolution of the Galactic GCs (Dinescu et al. 1997, 1999), we consider that R_i , which corresponds to the apocenter distance, is $2R_{d,mw}$ (where $R_{d,mw}$ is the stellar disk size of the Galaxy and 17.5 kpc in the present study), $f_v = 0.4$, and $\theta = 46.6^\circ$ are reasonable for the orbit of the dwarf. We however investigate models with different R_i , f_v , and θ owing to some observational uncertainties of the proper motion of NGC 1851.

Therefore the initial 3D position and velocity of the dwarf are set to be $(x, y, z) = (R_i \cos \theta, 0, R_i \sin \theta)$ and $(V_x, V_y, V_z) = (0, f_v v_c, 0)$, respectively. Although we have investigated models with a variety of different R_i , f_v , and θ for a given $M_{s,dw}$ (and $R_{s,dw}$), we mainly describe here the results of the “standard model” with $R_i = 35$ kpc and $f_v = 0.4$, and $\theta = 46.6^\circ$. Fig. 1 shows the orbit with respect to the Galaxy for four representative models with $R_i = 35$ kpc, $\theta = 46.6^\circ$, and $f_v = 0.2, 0.4, 0.6$, and 0.8 .

2.4 GC merging

Previous numerical simulations on GC evolution in *isolated* low-mass disk galaxies showed that GC merging in the central regions of the galaxies is possible (Bekki 2010a). GC merging processes however have not been extensively investigated for the NGC 1851’s host dwarf *interacting with the Galaxy*. Therefore, we investigate whether GC merging can occur in dwarfs before the complete destruction of the dwarfs by the Galaxy using the models with GCs, though we focus mainly on the transformation processes from nucleated dwarfs into GCs in the models with no GCs described in §2.1. Two GCs are represented by point-mass particles with masses of m_{gc} and are referred to as GC1 and GC2. The ini-

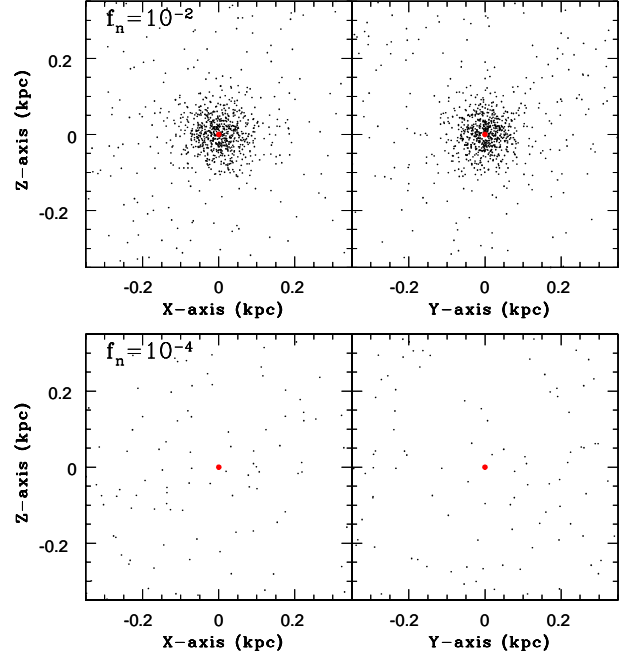


Figure 4. The final distributions of disk field stars (i.e., stellar envelope) around the stripped nuclei projected onto the x - z plane (left) and the y - z one (right) for the standard model (M1) with $f_n = 10^{-2}$ (upper two) and the comparative model (M5) with $f_n = 10^{-4}$. The central big red particle represents the location of the stellar nuclei of the dwarfs.

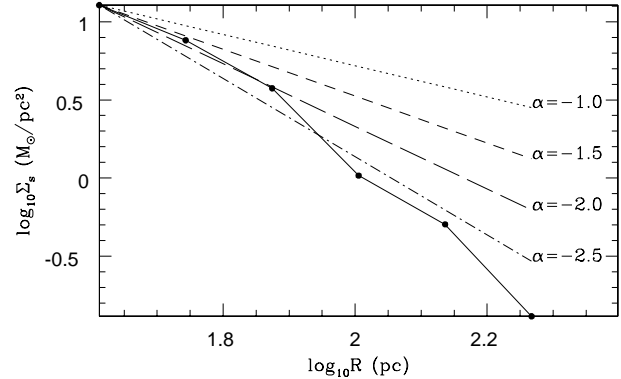


Figure 5. The final projected radial density profile of the stellar halo around the stripped nucleus in the standard model ($T = 11.3$ Gyr). For comparison, the power-law density profiles with the slopes $\alpha = -1.0, -1.5, -2.0$, and -2.5 are shown by dotted, short-dashed, long-dashed, and dot-dashed lines. The best-fit observational slope is -1.24 in the range of $60 - 300$ pc (Olszewski et al. 2009) and the observational data can also fit to the slopes of -0.58 and -1.9 within 1σ errors.

tial distances of GC1 and GC2 from the center of the host are denoted as r_1 and r_2 , respectively. The initial velocities of the GCs are chosen such that they are the same as those of disk field stars that are closest to the GC particles.

We follow the orbital evolution of the GCs and investigate whether the GCs can satisfy the following two physical conditions of GC merging at each time step: (i) the distance of the two (R_{gc}) is less than 10 pc and (ii) the total energy (E_t) is less than 0. Here $E_t = E_p + E_k$, where E_p

and E_k are the total potential energy and kinetic energy of a system composed *only* of the two GCs represented by point-mass particles, respectively. We estimate E_p and E_k based on m_{gc} , the distance between the two GCs (R_{gc}), the velocity difference between GC1 and GC2 (V_r) at each time step. We consider that if the above two conditions are satisfied, then the two GCs are regarded as “merged” and we thereby investigate the time scale of GC merging (t_m). The initial locations of two GCs are randomly chosen and the GCs are assumed to rotate around the host dwarf like disk field stars.

We first run models in which dynamical evolution of dwarfs with two GCs is followed until two GC merge. For these models, we estimate the time scale of GC merging for a given set of model parameters. We then run models in which merging of two GCs results in the formation of the new GC and the subsequent evolution of the GC in the dwarf is followed. For these models, the new GC just after GC merging has the total mass of GC1 and GC2 and the position and velocity that are exactly the same as those for the center of mass of the original two GCs. Table 1 summarizes the parameters for 33 representative models for which the results are discussed in the present paper. The models M1-M17 with no GCs are for investigating the dynamical evolution of nucleated dwarfs whereas the models MG1-9 are for estimating t_m . We investigate whether the merged GCs can sink into the nuclear region of their host dwarf and finally become stellar nuclei (or nuclear star clusters) by using the models MGM1-7.

We primarily consider “equal-mass” GC mergers, because this is consistent with some observations (e.g., Milone et al. 2008) which showed that the mass-ratio of two sub-populations in NGC 1851 is 45:55. Han et al. (2009) however suggested that the mass-ratio is 75:25. The latest results by Milone et al. (2009) are slightly different from those by Milone et al. (2008) and more consistent with those by Han et al. (2009). Given these possibly different mass-ratios derived from different observations, it would be important to investigate models in which two GCs have different initial masses. We consider that equal-mass GC merger models enable us to grasp essential ingredients of GC merging processes in dwarfs interacting with the Galaxy. Thus we briefly describe and discuss the results of the different initial GC mass models in the Appendix A.

2.5 Simulation set up

In order to simulate the long-term dynamical evolution of nucleated dwarfs with and without GCs for ~ 11 Gyr, we use the latest version of GRAPE (GRAVITY PipE, GRAPE-7), which is the special-purpose computer for gravitational dynamics (Sugimoto et al. 1990). We use our original GRAPE code (Bekki 2010a) which enables us to investigate both global dynamical evolution of galaxies and orbital evolution of GCs within them. The time integration of the equation of motion is performed by using 2nd-order leap-frog method with a time step interval of $\sim 0.01t_{dyn}$, where t_{dyn} is the dynamical time scale of a host dwarf. The total number of particles for the dark matter (N_{dm}) and the stellar envelope (N_s) used in the standard model is 200,000 and 200,000, respectively. It takes about 86 CPU hours for us to run the above standard model using one GRAPE7 (model 300E). We

have a limited amount of computations time for the present study and we need to run at least ~ 100 models with different model parameters in the present investigation. We therefore consider that the total number particle of 400,000 is reasonable (not numerically costly) that we can adopt for each model. We confirm that the results do not depend on N for $N \geq 400,000$.

The gravitational softening lengths (ϵ) for the dark matter particles and the stellar ones are denoted as $\epsilon_{dm,dw}$, $\epsilon_{s,dw}$, respectively. We determine ϵ for each of these components based on the half-number radius of the particles. The softening lengths for GCs and nucleus particle are assumed to be the same as $\epsilon_{s,dw}$. We consider that when two different components interact gravitationally, the mean softening length for the two components is applied for the gravitational calculation. For example, $\epsilon = (\epsilon_{dm,dw} + \epsilon_{s,dw})/2$ is used for gravitational interaction between dark matter particles and stellar ones in a dwarf. In the standard model, $\epsilon_{dm,dw}$ and $\epsilon_{s,dw}$ are set to be 128 pc and 20 pc, respectively. In the following, T in a simulation represents the time that has elapsed since the simulation started.

3 RESULTS

We first describe the formation processes of the stellar halo around NGC 1851 based on the results of the models (M1-18) in which two GCs are not included in §3.1. We then describe the results of the models with GCs (MG1-9) and those in which two GCs are allowed to merge with each other and thereby form new GCs (MGM1-5) in §3.2

3.1 Stellar halo formation

3.1.1 The standard model

Fig. 2 summarizes the orbital evolution of the nucleated dwarf around the Galaxy and the transformation processes of the dwarf into a naked nucleus (i.e., GC NGC 1851). As the dwarf passes by the pericenter distance, the stellar envelope of the dwarf is strongly influenced by the tidal field of the Galaxy so that stars in the dwarf can be efficiently stripped. As a result of this, the dwarf gradually loses their dark matter halo and stars and the size of the stellar disk becomes significantly smaller ($T = 2.8$ Gyr). Although almost all of the dark matter has been tidally stripped from the dwarf by $T = 8.5$ Gyr, a minor fraction of the disk field stars can remain trapped by the gravitational potential of the stellar nucleus and thus surround the nucleus. Finally the total masses of the dark matter halo and the stellar envelope within 200 pc from the nucleus become $4.5 \times 10^3 M_\odot$ and $3.3 \times 10^5 M_\odot$, respectively ($T = 11.3$ Gyr). The stars around the stripped stellar nucleus can be observed as a stellar halo around NGC 1851. The stripped stars can finally form numerous long tidal streams that follow the orbit of the dwarf.

Fig. 3 clearly shows that the dark matter halo with the cored radial density profile can be efficiently stripped so that the total mass within the central 500 pc ($M_{dm,500pc}$) can decrease dramatically. As a result of this, $M_{dm,500pc}$ becomes $8.7 \times 10^5 M_\odot$, $1.8 \times 10^4 M_\odot$, and $4.5 \times 10^3 M_\odot$, at $T = 2.8$ Gyr, 8.5 Gyr, and 11.3 Gyr, respectively. The derived $M_{dm,500pc}$

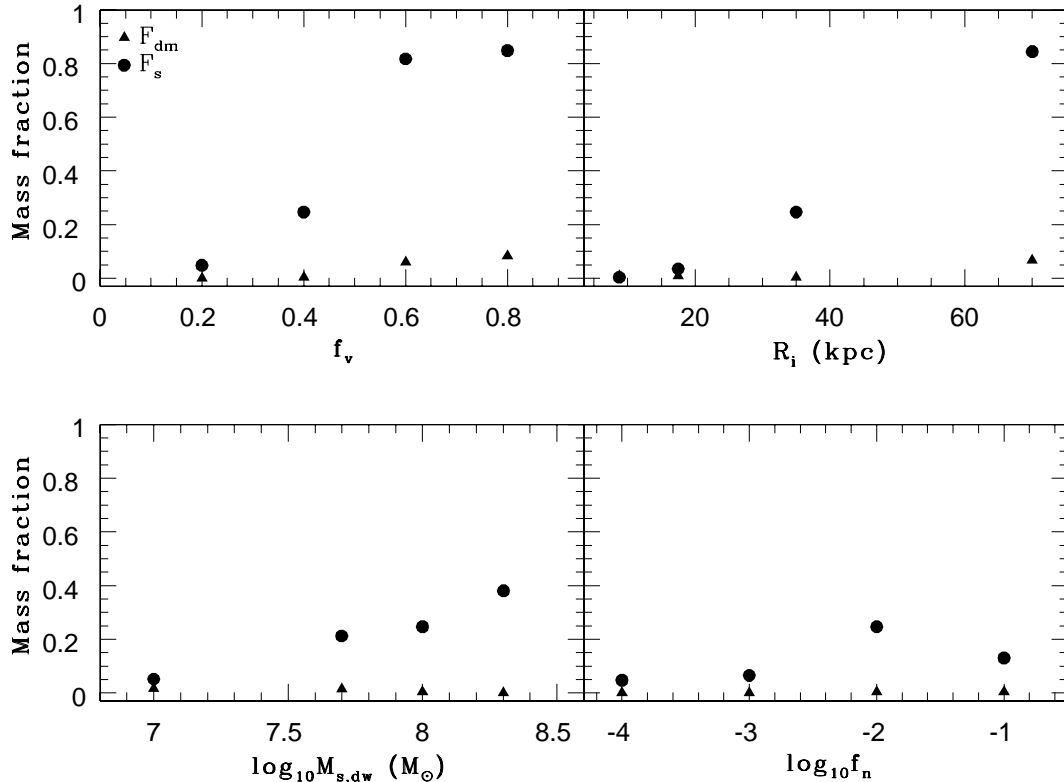


Figure 6. The dependences of dark matter mass fractions (F_{dm} ; filled triangles) and stellar halo ones (F_s ; filled circles) in stripped nuclei on the velocity factor (f_v , upper left), the initial distance of the dwarf from the Galactic center (R_i , upper right), the initial stellar mass of the dwarf ($M_{s,\text{dw}}$, lower left), and the initial nucleus mass fraction (f_n , lower right). For each of these plots, only the indicated parameter (e.g., f_v in the upper left panel) is changed whereas other parameters are fixed.

means that the naked nucleus (=GC NGC 1851) is highly likely to be observed as a GC with no dark matter halo, though it can have a tiny fraction of dark matter. Inspection of Figs. 2 and 3 demonstrate that even if NGC 1851 is formed from the destruction of a nucleated dwarf embedded in a massive dark matter halo, it can have no/little dark matter within it. It should be here stressed that if the cuspy NFW dark matter halo is adopted for the model of a nucleated dwarf, then the dwarf is less likely to be destroyed by the tidal field of its host environment (see Bekki et al. 2003).

Fig. 4 shows the final distributions of the stellar halos formed around the stripped nuclei for the standard with $f_n = 0.01$ and for the comparative model M5 with a less massive nucleus and $f_n = 10^{-4}$. The stellar halo in the standard model shows a symmetric and spherical distribution with no tidal tails whereas the stripped nucleus can hardly keep disk field stars in the model M5. This clear difference demonstrates that strong gravitational fields of stellar nuclei are important for the formation of stellar halos around the nuclei during the destruction nucleated dwarfs by the Galaxy. Fig. 5 shows that the projected density distribution of the stellar halo in the standard model can provide better fit to the power-law profile with the slope (α) of ~ -2 for $R \leq 80$ pc. The projected profile has a steeper profile ($\alpha \sim -2.5$) in the outer part ($80 < R < 250$ pc) of the stripped nucleus: there is a significant difference between models and observations in the outer part of the halo. The derived α is somewhat flatter in comparison with those derived by previous simulations in Küpper et al. (2010). The

derived $\alpha \sim -2$ for $R \leq 80$ pc is slightly steeper than the observed $\alpha \sim -1.24$ (Olszewski et al. 2009), though the observational results appear to fit to a range of values from $\alpha \sim -2$ to -0.6 (1σ limits). The derived flat spatial distribution of the stellar halo in the present study can be regarded as qualitatively similar to (yet slightly steeper than) those observed by Olszewski et al. (2009). Fine tuning of the models could result in an improved fit.

3.1.2 Parameter dependence

Whether or not nucleated dwarfs can be transformed into GCs (i.e., stripped nuclei) within ~ 10 Gyr depends strongly on R_i and f_v for a given $M_{s,\text{dw}}$. The ratios of dark matter masses to total masses (including dark matter, stellar envelopes, and nuclei) within 200 pc of stripped nuclei (F_{dm}) depend only weakly on model parameters. The mass-ratios of stellar halos around stripped nuclei (F_s) are different in models with different $M_{s,\text{dw}}$ and f_n (initial value) for a given orbit. In Figures 6, we illustrate the derived dependences on model parameters. We find the following:

(i) The dwarfs in the models with larger f_v ($=0.6$ and 0.8) are much less strongly influenced by the Galaxy owing to the more circular orbits with larger pericenter distances. morphological transformation As a result of this, from nucleated dwarfs into GCs does not occur and f_n are large in these models. These results imply that highly radial orbits of the dwarfs are necessary for nucleated dwarfs to be transformed into GCs within ~ 10 Gyr. The models with lower

f_v show smaller F_s and smaller total masses of stellar halos around stripped nuclei (M_{halo}). For example, the model with low $f_v = 0.2$ shows a faint stellar halo with $f_n = 0.05$ and $M_{\text{halo}} = 5.1 \times 10^4 M_\odot$ around the stripped nuclei.

(ii) The models with $R_i = 70 \text{ kpc}$ (i.e., large apocenter distances) do not show transformation from nucleated dwarfs into GCs for $f_v = 0.4$ and $M_{s,dw} = 10^8 M_\odot$. This result suggests that GCs currently located in the outer halo of the Galaxy ($\sim 70 \text{ kpc}$) are less likely to be formed from nuclei of nucleated dwarfs. Both F_s and M_{halo} are smaller for models with smaller R_i . For example, $f_n = 0.004$ and $M_{\text{halo}} = 4.0 \times 10^3 M_\odot$ in the model with $R_i = 0.5$. This suggests that if GCs originate from nuclei of nucleated dwarfs orbiting the inner regions of the Galaxy, the faint stellar halos around GCs can be observationally difficult to detect separately from stars inside GCs.

(iii) Nucleated dwarfs with $M_{s,dw} \leq 2 \times 10^8 M_\odot$ can be transformed into GCs for $R_i = 35 \text{ kpc}$ and $f_v = 0.4$. The final f_n is larger for dwarfs with larger $M_{s,dw}$, because more massive nuclei in more massive dwarfs can trap a larger fraction of disk field stars. These results imply that the most massive GCs will have higher surface brightness stellar halos. Therefore, stellar halos are more likely to be detected around the more massive GCs.

(iv) The models with larger f_n show larger F_s for $f_n \leq 0.01$, because more massive nuclei can continue to trap their surrounding stars in their deeper gravitational potential wells during destruction of their host dwarfs. Although the model with $f_n = 10^{-1}$ shows F_s slightly smaller than that in the model with $f_n = 10^{-2}$, M_{halo} is larger in the model with $f_n = 10^{-1}$ ($M_{\text{halo}} = 1.5 \times 10^6 M_\odot$). The models with $f_n = 10^{-2}$ and 10^{-3} show $M_{\text{halo}} = 7.0 \times 10^3 M_\odot$ and $5.1 \times 10^2 M_\odot$, respectively.

(v) The final M_{halo} can be rather small in the LSB models with highly eccentric orbits, because larger fractions of stars in the dwarfs can be stripped by the tidal field of the Galaxy. For example, the LSB models with $f_v = 0.4$ (M14) and 0.2 (M15) show $M_{\text{halo}} = 3.3 \times 10^4 M_\odot$ and $4.2 \times 10^3 M_\odot$, respectively. The low-mass models ($M_{s,dw} = 10^7 M_\odot$) with $f_v = 0.4$ (M16) and 0.2 (M17) show $M_{\text{halo}} = 1.9 \times 10^4 M_\odot$ and $2.1 \times 10^3 M_\odot$, respectively. As discussed later, these models can be in better agreement with the observationally estimated M_{halo} in NGC 1851.

(vi) All models which show transformation of nucleated dwarfs into GCs show $F_{\text{dm}} < 0.01$, which implies that if GCs originate from nuclei of dwarfs, they can have no/little dark matter within them. This is mainly because cored dark matter halos are adopted in the present study. The present results do not depend strongly on initial inclination angles of stellar disks (θ_d and ϕ_d) in dwarfs.

3.2 GC merging within dwarfs

Fig. 7 shows the time evolution of distances between GC1 (GC2) and its host (labeled as “GC1-dwarf distance”) dwarf and between GC1 and GC2 (“GC1-GC2 distance”) for the model MG1 in which model parameters for the host are exactly the same as those adopted in the standard model M1. The formation of a new GC by merging of two GCs is *not* included in this model MG1. Clearly, the two GCs can quickly sink into the nuclear region of the host owing to dynamical friction against disk field stars so that they can

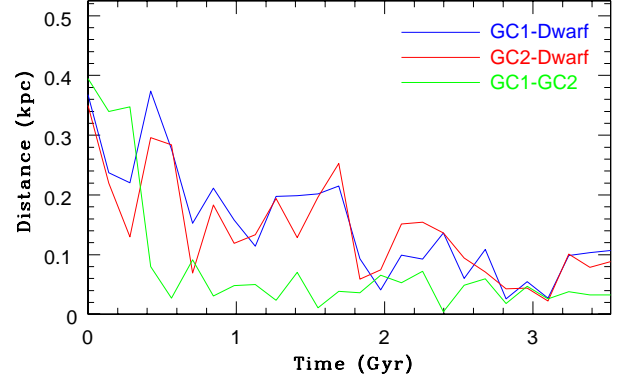


Figure 7. Time evolution of distances between GC1 and its host dwarf (blue), GC2 and the dwarf (red), and GC1 and GC2 (green) in the model MG1 with $r_1 = r_2 = 0.2 R_{s,dw}$.

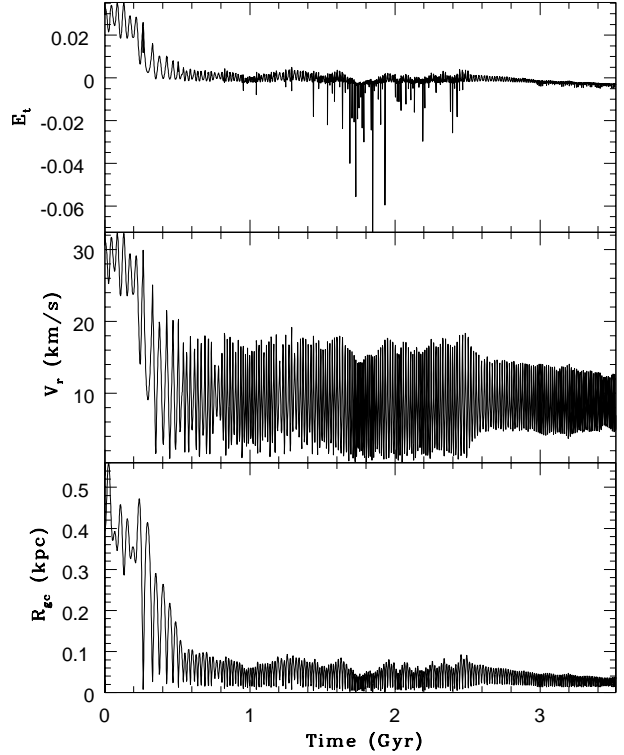


Figure 8. Time evolution of the total energy of two GCs (E_t , top), the relative velocity (V_r , middle), and the mutual distance (R_{gc} , bottom) in the model MG1.

finally be located within the central 100 pc. The GC1-GC2 distance can become well less than 100 pc within the first 1 Gyr of their evolution, though their distance is initially as large as $\sim 500 \text{ pc}$.

Fig. 8 shows that as the two GCs get close to each other ($R_{gc} < 100 \text{ pc}$), both E_t and V_r become significantly smaller. The two can have $R_{gc} < 10 \text{ pc}$ and $V_r < 10 \text{ km s}^{-1}$ within the first $\sim 1 \text{ Gyr}$ so that they can have $E_t < 0$: the two GCs are in a strongly bound orbit and can merge with each other to form a new GC in the central region of the host. Orbital evolution of GCs and merging processes

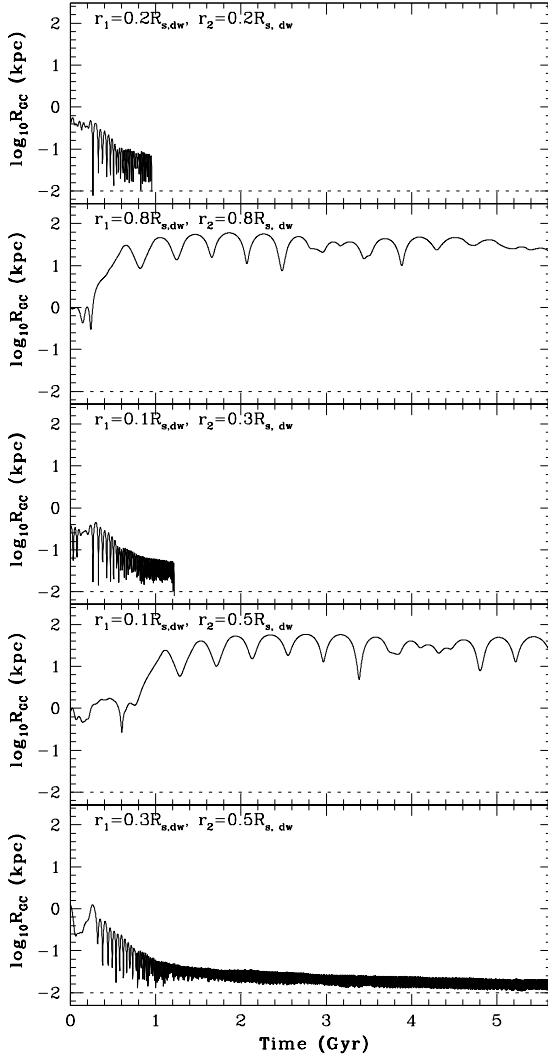


Figure 9. Time evolution of R_{gc} of two GCs for models with $r_1 = r_2 = 0.2R_{s,dw}$ (top) $r_1 = r_2 = 0.8R_{s,dw}$ (second from the top) $r_1 = 0.1R_{s,dw}$ and $r_2 = 0.3R_{s,dw}$ (third from the bottom) $r_1 = 0.1R_{s,dw}$ and $r_2 = 0.5R_{s,dw}$ (second from the bottom) and $r_1 = 0.3R_{s,dw}$ and $r_2 = 0.5R_{s,dw}$ (bottom). The R_{gc} evolution is shown for $T < t_m$ in each model, where t_m is the time when two GCs merge.

of two GCs depend strongly on where the GCs are born in the host. Fig. 9 shows evolution of R_{GC} for different r_1 and r_2 and thereby describes when GC merging is possible in each model. It is clear that if $r_1 = r_2$ and if $r_1 \sim 0.2R_{s,dw}$, then GC merging can occur in their host within ~ 1 Gyr for $m_{gc} = 5 \times 10^5 M_\odot$. GCs initially in the outer part of the host ($r_1 \sim 0.8R_d$) can be tidally stripped by the Galaxy so that they can finally become halo GCs: GC merging is highly unlikely for these GCs. Fig. 9 also shows that GCs with initially different r_1 and r_2 can merge within ~ 1 Gyr, as long as both r_1 and r_2 are smaller ($\leq 0.3R_{s,dw}$). Such merging of GCs originating from different regions of their host could be responsible for the formation of GCs with abundance spreads in heavy elements, as discussed later.

Fig. 10 shows that the time evolution of R_{gc} for two GCs depends strongly on m_{gc} for a given model of their

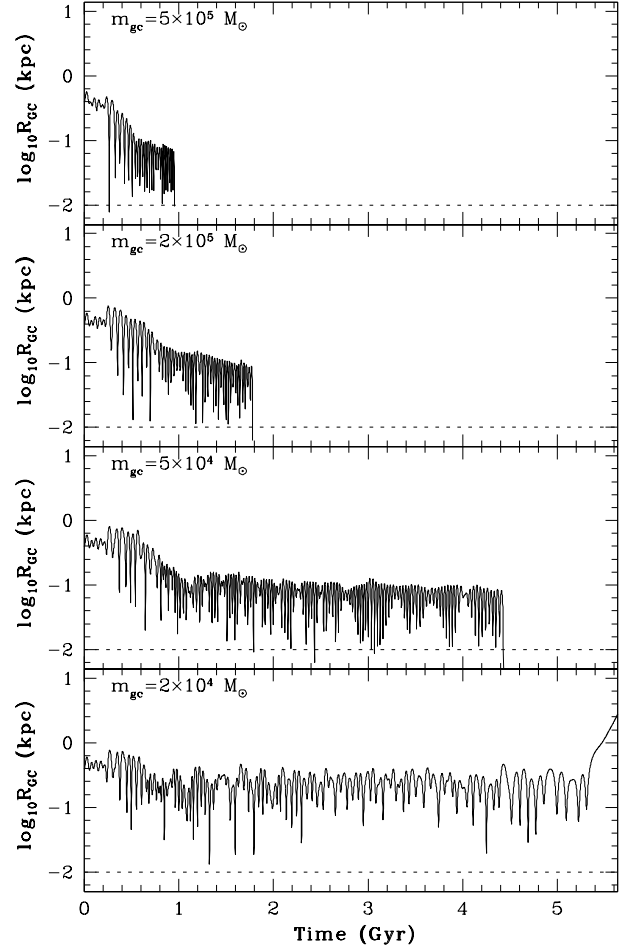


Figure 10. The same as Figure 9 but for models with different m_{gc} : $5 \times 10^5 M_\odot$ (top), $2 \times 10^5 M_\odot$ (second from the top), $5 \times 10^4 M_\odot$ (second from the bottom), and $2 \times 10^4 M_\odot$ (bottom)

host. The time scale of GC merging is longer for smaller m_{gc} , because it takes a longer time for GCs with smaller m_{gc} to sink into their host's central region, where GC merging can occur: the time scale of dynamical friction can be inversely proportional to m_{gc} . This means that GCs with smaller m_{gc} are more likely to be tidally stripped from their host by the Galactic tidal field before they merge with one another to become new GCs within the host. This also means that if the present GCs have multiple stellar populations with abundance spreads in heavy elements, they are likely to be quite massive GCs, in agreement with current observations.

Fig. 11 shows the final distribution of stars around merged GCs in the model MGM1 in which merging of GCs with $m_{gc} = 5 \times 10^5 M_\odot$ and the evolution of merged GCs are included. For comparison, the model MGM6 in which each of the two GCs has a smaller mass ($m_{gc} = 2 \times 10^5 M_\odot$) is shown in Fig. 11. Although the two GCs in the model MGM1 are initially in the central region of its host ($r_1 = r_2 = 0.2R_{s,dw}$), they are initially not nuclear star clusters. While the host is being disrupted by the Galaxy, the two GCs can merge with each other and then sink into the nuclear region. The new cluster can finally become a new nuclear cluster (or a stellar galactic nucleus) surrounded by disk field stars. After the complete disruption of the host, the naked nucleus can

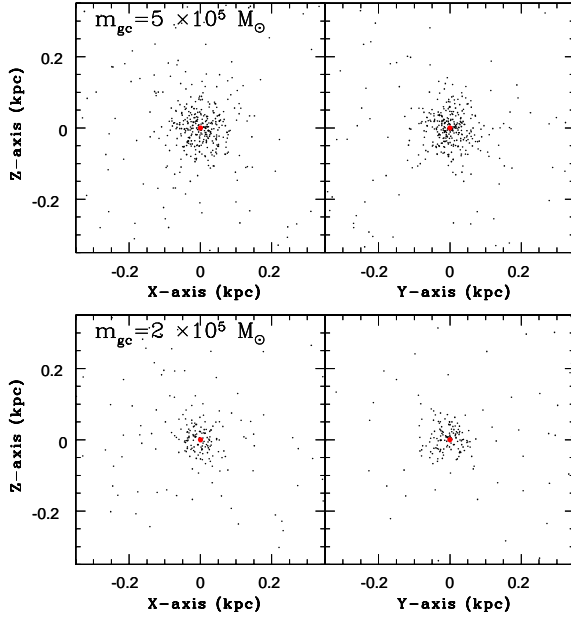


Figure 11. The final distributions of disk field stars (i.e., stellar envelope) around the merged GCs projected onto the x - z plane (left) and the y - z one (right) for the model MGM1 with $m_{gc} = 5 \times 10^5 M_\odot$ (upper) and MGM6 with $m_{gc} = 2 \times 10^5 M_\odot$ (lower). The central big red particle represents the location of the merged GC in each model.

appear as a GC with a diffuse stellar halo. Although the numerical method for GC merging would be simplified in this model MGM1, the results in Fig. 10 self-consistently demonstrate that NGC 1851 with a stellar halo can be formed as a result of GC merging in the central region of its host during the disruption of the host by the Galaxy.

The naked nucleus (i.e., merged GCs) has no dark matter and $M_{halo} = 1.5 \times 10^5 M_\odot$, which is smaller by a factor of ~ 3 than M_{halo} derived for the standard model. Other models (MGM2, 3, 4, and 5) with $m_{gc} \geq 5 \times 10^5 M_\odot$ with different r_1 and r_2 show $M_{halo} \sim 10^5 M_\odot$. The final M_{halo} around stripped nuclei depends on m_{gc} such that M_{halo} can be smaller in smaller m_{gc} . Fig. 11 shows that the stellar halo appears to be more diffuse and compact in the model with $m_{gc} = 2 \times 10^5 M_\odot$ ($M_{halo} = 6.9 \times 10^4 M_\odot$). The final M_{halo} depend on $M_{s,dw}$ and $R_{s,dw}$ such that M_{halo} can be smaller in smaller $M_{s,dw}$ and larger $R_{s,dw}$. For example, the LSB model MGM7 with $M_{s,dw} = 10^7 M_\odot$ and $R_{s,dw} = 2.8$ kpc shows $M_{halo} = 2.7 \times 10^4 M_\odot$.

Thus we have demonstrated that the merging of two GCs with $m_{gc} \sim 5 \times 10^5 M_\odot$ (and $2 \times 10^5 M_\odot$) in a host dwarf could be the progenitor for NGC 1851. We reiterate that these simulations, and therefore results, make no assumption on the ages and/or chemical abundances of the two merging GCs. If two GCs have slightly different ages and s -process abundances, then the merged GC can show chemical abundances consistent with the NGC 1851 observations. We have so far assumed that massive progenitor GCs could be formed in the early history of the host with a smaller mass of $\sim 10^9 M_\odot$. It is not observationally clear whether such massive GCs can form in a dwarf galaxy at a high redshift.

However Georgiev et al. (2009) found that old massive GCs exist in the present-day dwarfs, which implies that massive GC formation could be possible in high-redshift dwarfs. In a future study, we plan to investigate whether two GCs with different ages and chemical abundances can be formed in a dwarf interacting with the Galaxy using more sophisticated chemodynamical simulations.

4 DISCUSSION

4.1 The total mass and stellar populations of the stellar halo around NGC 1851

The present study did not consider the evolution of individual stars with different masses through supernova explosion and mass loss in evolved stars. Therefore the derived M_{halo} in NGC 1851 in simulations corresponds to the initial total mass of the stellar halo around NGC 1851 (not the present mass). Olszewski et al. (2009) estimated M_{halo} ($\sim 500 M_\odot$) by assuming that the typical mass of the observed main-sequence stars in the halo around NGC 1851 is $0.5 M_\odot$. In order to make a more self-consistent comparison between the observed and simulated M_{halo} , we here estimate the mass fraction (F_{main}) of main-sequence stars with masses ranging from $m_{m,l}$ (lower mass cutoff for the observed main-sequence stars) to $m_{m,u}$ (upper mass cutoff) in a stellar population. Only stars brighter than $V = 25$ mag are observed in Olszewski et al. (2009), and $V = 25$ mag corresponds to $\sim 0.45 M_\odot$ ($\sim 0.41 M_\odot$) for $[Fe/H] = -0.9$ and $[\alpha/Fe] = 0.3$ ($[Fe/H] = -1.5$ and $[\alpha/Fe] = 0.3$) if the Yonsei-Yale isochrones for an age of 10 Gyr are adopted. Therefore fainter stars with masses significantly smaller than $0.45 M_\odot$ would not be included in the observational estimation of M_{halo} . We here estimate F_{main} for different $m_{m,l}$ and a fixed $m_{m,u}$ ($= 0.5 M_\odot$). We adopt an initial mass function (IMF) with a slope of α_{imf} , a lower mass cutoff of $0.1 M_\odot$, and an upper mass cutoff of $100 M_\odot$.

If we adopt $\alpha_{imf} = -2.35$ (i.e., canonical Salpeter IMF), then $F_{main} = 0.24$ for $m_{m,l} = 0.2 M_\odot$ and 0.05 for $m_{m,l} = 0.4 M_\odot$. If $m_{m,u} = 0.4 M_\odot$, then F_{main} is 0.19 for $m_{m,l} = 0.2 M_\odot$, which means that F_{main} does not depend so strongly on $m_{m,u}$ for a reasonable range of $m_{m,u}$. If we adopt a top-heavy IMF with $\alpha_{imf} = -1.35$, then $F_{main} = 0.014$ for $m_{m,l} = 0.2 M_\odot$ and 0.004 for $m_{m,l} = 0.4 M_\odot$. The initial total mass of the stellar halo around NGC 1851 is the observed M_{halo} divided by F_{main} . Therefore, the initial halo mass is $2.0 \times 10^3 M_\odot$ for $\alpha_{imf} = -2.35$ (i.e., canonical Salpeter IMF) and $m_{m,l} = 0.2 M_\odot$ and is $1.0 \times 10^4 M_\odot$ for $\alpha_{imf} = -2.35$ and $m_{m,l} = 0.4 M_\odot$. For a top-heavy IMF with $\alpha_{imf} = -1.35$ and $m_{m,l} = 0.4 M_\odot$, the initial halo mass can be as large as $\sim 1.3 \times 10^5 M_\odot$. If we adopt the rather shallow IMF slope (-1.3) for stellar masses of $0.08 - 0.5 M_\odot$ in the Kroupa's IMF, the initial halo mass can be significantly larger than those derived for the above canonical IMFs. The present models that have a canonical mass-size relation (i.e., $C_d = 8.75$) and show the formation of naked nuclei have shown M_{halo} of $[0.5 - 3.3] \times 10^5 M_\odot$ for $M_{s,dw} = 10^8 M_\odot$, $M_n = 10^6 M_\odot$ (or $f_n = 0.01$), $R_i = 35$ kpc, and $f_v \leq 0.4$. In the present study, only the LSB models (i.e., $C_d = 13.8$) with $M_{s,dw} = 10^7 M_\odot$, $M_n = 5 \times 10^5 M_\odot$ (or $f_n = 0.05$), $R_i = 35$ kpc, and $f_v = 0.2$ have shown M_{halo}

that can be as small as the above possible minimum initial mass ($= 2.0 \times 10^3 M_\odot$) of the stellar halo around NGC 1851. For example, the LSB model M14 with $M_{s,dw} = 10^7 M_\odot$, $f_n = 0.005$, $R_{s,dw} = 2.8$ kpc and $R_i = 35$ kpc, and $f_v = 0.2$ shows $M_{halo} = 4.2 \times 10^3 M_\odot$. These results suggest that if NGC 1851 originates from a stellar nucleus of a host dwarf, then the dwarf needs to have a very low surface brightness (or “darker” galaxy) and an highly eccentric orbit ($e \sim 0.8$) to explain the observed small M_{halo} . Such dwarf galaxies with highly eccentric orbits in a galaxy-scale halo can be seen in recent cosmological N-body simulations (e.g., Wetzel 2011).

GC merging can form diffuse stellar halos around the remnant GCs (see Fig. 1 in Bekki et al. 2004). If the remnants of GC mergers can sink into nuclear regions of their hosts and finally become stellar nuclei (or nuclear star clusters), then the merged GCs can trap surrounding field stars only after they become the nuclear GCs, as shown in the present study. Therefore, if nucleated dwarfs are transformed into GCs by tidal destruction of the dwarfs by the Galaxy, then the final GCs are likely to contain at least three different stellar populations with different abundances in heavy elements: the two are from original two GCs that merged with each other and the remaining ones are from field stars surrounding stellar nuclei. If two GCs that are progenitors for NGC 1851 were formed in the nuclear region of their host, then there would be small differences in abundances in heavy elements between the two GCs and stars in the nuclear region. If this is the case, then the possible three stellar populations in the stellar halo (and NGC 1851 itself) would not show abundance spreads in heavy elements. Olszewski et al. (2009) already revealed two stellar populations in the color-magnitude diagram of stars in the halo of NGC 1851, though they did not discuss the presence of the third population. We suggest that GC merging and trapping field stars by a stellar nucleus in the NGC 1851’s host dwarf can be responsible for the origin of the observed possible multiple populations in the halo around NGC 1851.

4.2 Other possible formation mechanisms for stellar halo formation around GCs

Although we have suggested that diffuse stellar halos around GCs are fossil evidence for the formation of GCs in the central regions of nucleated dwarfs, there could be other formation mechanisms for the halos. Recent theoretical studies on the origin of multiple stellar population in GCs have shown that (i) GCs were originally much more massive than the present GCs, (ii) GCs originally have two generations of stars with the second generation (SG) being formed from gas from AGB or massive stars of the first generation (FG), and (iii) the first generation of stars have more extended stellar distributions (e.g., D’Ercole et al. 2008; Bekki 2010b, 2011). They have shown that FG stars with much more extended distributions ($R \sim 200$ pc) can be stripped quite efficiently by the tidal field of the Galaxy (D’Ercole et al. 2008) and by the host dwarf galaxy (Bekki 2011). During the tidal stripping, the faint FG population could be regarded as stellar halos around the more compact second generations of stars. Therefore, it is possible that the observed diffuse stellar halo of NGC 1851 were stars that initially belonged to FG of an originally massive GC. If this is the case, then the halo

should have stars with chemical abundances consistent with those of FG (e.g., N-normal, C-normal stars) predicted in the above theoretical studies.

This “FG scenario” would have some potential problems as follows. Firstly, it is unclear why only some of GCs have been so far observed to have diffuse stellar halos around them (though this could be due to an observational bias). In other words, it is unclear why FG stars have been completely stripped for most GCs but not for NGC 1851. Some GCs like NGC 1851 would be massive enough to keep FG stars against tidal stripping of the stars by the Galaxy. Secondly, given the predicted short time scale ($< a$ few Gyr) of stripping of FG stars (D’Ercole et al. 2008; Bekki 2011), it would be hard to understand how a significant fraction of FG stars have continued to surround NGC 1851 for more than 10 Gyr. The observed extension (~ 500 pc) and radial density profile of the halo would be difficult to be explained by the FG scenario, which is based on a nested cluster with the size of ~ 100 pc. Thus it is possible, yet unlikely, that the FG scenario can self-consistently explain the observed properties of the stellar halo around NGC 1851. As discussed by Olszewski et al. (2009), the lack of tidal tails within ~ 500 pc from the center of NGC 1851 is inconsistent with a scenario in which the halo stars originate from stars stripped from NGC 1851 itself. The rather steep profiles of the stripped stars from GCs in Küpper et al. (2010) imply that the stellar halo of NGC 1851 is unlikely to originate from NGC 1851 itself.

If NGC 1851 really originates from a stellar nucleus of a defunct dwarf galaxy, and if the nucleus was formed *in situ*, then chemical abundances of the diffuse stellar halo around NGC 1851 should be similar to those of stars currently within NGC 1851. If the nucleus was formed from merging of two that were initially located in the central regions of the host dwarf, then there could be some differences in chemical abundances between NGC 1851 and its stellar halo. The abundance differences would depend both on where GCs were formed within the host dwarf and on the radial abundance gradient within the dwarf. Thus future spectroscopic observations on chemical abundances of the stellar halo around NGC 1851 will provide important constraints on the origin of the stellar halo.

4.3 Radial metallicity gradients of dwarfs as an origin of dispersions in heavier elements of GCs

For GC merging to be really responsible for two distinct stellar populations with different chemical abundances, then the host dwarf needs to have a radial metallicity gradient, because merger progenitor GCs are highly likely to form at different radii within the dwarf. Recent observations have revealed that nearby dwarf elliptical (Crnojević et al. 2010) and dwarf spirals (Hidalgo-Gómez et al. 2010) have negative abundance gradients in [Fe/H] and oxygen with ~ -0.2 dex kpc^{-1} , though it is unclear whether there are also age gradients in these galaxies. Two GCs with different r_1 and r_2 (distances from the center of its host) in the present standard model can quickly merge with each other to form a new GC, if both r_1 and r_2 are less than ~ 500 pc. This means that if the host dwarf has a radial abundance gradient similar to the above observed one, then NGC 1851 formed from GC

merging is expected to have a spread in $[\text{Fe}/\text{H}] \sim -0.1$ dex. Recently, Carretta et al. (2010) have reported the presence of a dispersion in $[\text{Fe}/\text{H}]$ ($\sigma \sim 0.07$) between the two distinct stellar populations in NGC 1851. The similarity between the expected σ from GC merging and the observed one implies that the observed dispersion (Carretta et al. 2010) could be due to the radial metallicity gradient of the defunct host dwarf of NGC 1851. Thus we suggest that if GC merging is a key formation process of GCs, then the radial metallicity gradients of their host dwarf can determine to what extent the merged GCs can show abundance spreads in heavier elements. It should be stressed that two merging GCs are likely to form at different epochs in their host dwarf and thus the observed two subpopulations of NGC 1851 could have different ages, and thus different chemical enrichment histories.

Recent observational studies have revealed that some of the Galactic GCs (e.g., NGC 2419 and M22) show small yet real spreads in heavier elements (e.g., Marino et al. 2009, 2011; Da Costa et al. 2009; Cohen et al. 2010). The origin of the observed spread in NGC 2419 is suggested to be due to a massive host galaxy that previously contained NGC 2419 in its center and could retain stellar ejected from supernova for further star formation (Cohen et al. 2010). The present study suggests that host dwarf galaxies are important for the formation of these GCs, because (i) low velocity dispersions of the dwarfs allow GCs to merge quickly and (ii) there can be GCs that form in different regions within dwarfs and thus have different chemical abundances. Massive dark matter halos surrounding the host dwarfs would have played a vital role in retaining stellar eject from massive OB stars and supernova and thus in forming stars with different chemical abundances in different regions. As shown in our previous simulations (Bekki et al. 2003; Bekki & Freeman 2003; Bekki & Chiba 2004), dark matter halos of dwarfs can be almost completely stripped by their environments during transformation from nucleated dwarfs into GCs and UCDs. Thus, although dark matter halos of dwarfs would have played a decisive role in the formation of GCs with abundance spreads in heavy elements (e.g., NGC 2419), they are very hard to be detected in GCs directly by observations.

5 CONCLUSIONS

We have numerically investigated (i) the dynamical evolution of nucleated dwarfs orbiting around the Galaxy and (ii) the merging of two GCs in dwarfs in order to understand the entire formation history of NGC 1851. NGC 1851's host galaxy is modeled as a small bulge-less disk galaxy that has a stellar nucleus (or nuclear star cluster) and is embedded in a cored dark matter halo. The host is assumed to have two GCs represented by point-mass particles in some models so that physical processes of GC merging can be investigated in detail. The main results are summarized as follows.

(1) The host dwarf disk galaxy of NGC 1851 can be almost completely destroyed within ~ 10 Gyr by the strong tidal field of the Galaxy for a reasonable orbital model of the dwarf. Although the dark matter halo and the stellar disk of the dwarf can be almost completely stripped from the dwarf, a tiny fraction of the disk field stars can remain trapped by the nucleus. As a result of this, the stripped nucleus and the surrounding field stars can be observed as a GC and its

low surface-brightness stellar halo, respectively. The mass fraction of dark matter (F_{dm}) in NGC 1851 is typically less than 10^{-2} , which means that NGC 1851 can have little dark matter even if it originates from the central region of a dwarf galaxy dominated by dark matter.

(2) The simulated stellar halo around NGC 1851 has a symmetric and spherical distribution within 200 pc from the center of NGC 1851. The halo does not show tidal tails and it has a projected radial density profile with the power-law slope α of ~ -2 in the inner 80 pc and steepens beyond that radius. The derived slope is steeper than the best-fit one in observations (~ -1.2), though the observational data can also fit to $\alpha \sim -0.58$ and ~ -1.9 within the 1σ errors. The mass fraction of the halo (F_s) is typically ~ 0.1 and depends strongly on R_i , f_v , and $M_{s,\text{dw}}$. F_s can be as small as 10^{-3} (or $M_{\text{halo}} \sim 10^3 M_\odot$) in LSB models. These results imply that the observed mass fraction of the diffuse stellar halo around NGC 1851 can give constraints on the orbit and the mass/size of the defunct host dwarf.

(3) For the NGC 1851's host dwarf to be transformed into a GC with a stellar halo within ~ 10 Gyr, the host needs to have certain ranges of R_i and f_v for a given $M_{s,\text{dw}}$. For example, models with $f_v \geq 0.6$ for $R_i = 2R_{d,\text{mw}}$ do not show naked nuclei that can be observed as NGC 1851 whereas the models with $f_v \leq 0.2$ show fainter stellar halos ($F_s \ll 0.1$). Also, the nucleated dwarfs in the models with $R_i \geq 4R_{d,\text{mw}}$ cannot be destroyed by the Galactic tidal field for $0.2 \leq f_v \leq 0.8$.

(4) Two equal-mass GCs initially in the NGC 1851's host can merge with each other to form a new GC in the central region of the host within ~ 1 Gyr, if each of the two GCs has a mass of $\sim 5 \times 10^5 M_\odot$ and if they are located at $R \approx 0.2R_{s,\text{dw}}$. Merging between two GCs (GC1 and GC2) with different initial distances from the host's center (r_1 and r_2 , respectively) can occur within a few Gyr, though the timescale of GC merging depends strongly on r_1 and r_2 for a given m_{gc} . GC merging with different initial GC masses can also occur within ~ 1 Gyr if the total mass of the two GCs is as large as $10^6 M_\odot$. The merging timescale of two GCs can be significantly longer than ~ 1 Gyr for rather small GC mass-ratios (< 0.05). Given that interstellar gas at different R might well have different chemical abundances in the host (owing to a possible radial metallicity gradient), these results imply that the observed abundance spreads in s -process elements for NGC 1851 can be closely associated with GC merging in the central region of the host.

(5) Merged GCs can quickly sink into the central region of its host owing to dynamical friction and finally become a nuclear star cluster (or stellar nucleus) surrounded by disk field stars. However, not all merged GCs can become nuclear clusters surrounded by field stars. Merged GCs with low masses cannot continue to trap field stars in their gravitational potentials owing to their low masses, even if they become nuclear clusters. These imply that not all of GCs that were formed from GC merging within their host dwarfs can show diffuse stellar halos.

(6) GCs initially located in the outer regions of the host dwarf ($R > 0.6R_{s,\text{dw}}$) and those with lower masses ($m_{\text{gc}} < 5 \times 10^4 M_\odot$) are more likely to be stripped to become the Galactic halo GCs without GC merging during the orbital evolution of the host. Only more massive GCs formed initially in the inner regions of the host can merge

with each other to become nuclear GCs (i.e., stellar nuclei) before the host can be completely destroyed by the Galactic tidal field. These results imply that more massive GCs are more likely to show abundance spread even in *s*-process and heavy elements.

(7) If NGC 1851 originates from the stellar nucleus of a host dwarf, and if the nucleus was formed from GC merging, the stellar halo is highly likely to contain at least three different stellar populations with different heavy and *s*-process elements. The three populations are from two GCs and field stars in the nuclear region of the host dwarf. The host needs to be a dwarf with a very low surface brightness ($M_{s,dw} \sim 10^7 M_\odot$ and $R_{s,dw} \sim 1$ kpc) and a highly eccentric orbit ($e \sim 0.8$), if the total mass of the stellar halo around NGC 1851 is $\sim 10^3 M_\odot$.

Thus we have demonstrated that transformation from nucleated dwarfs into GCs, GC merging within dwarfs, and nucleus formation by GC merging are all possible in a self-consistent manner. The present study suggests that the formation processes of NGC 1851 are different from those of normal GCs in the following two points: (i) it experienced GC merging and (ii) it was once located in the nuclear region of its host dwarf. The present study suggests that some of the Galactic GCs, such as NGC 5694 with a diffuse stellar halo, can be formed in a similar way as NGC 1851. Furthermore the present study predicts that a larger fraction of GCs at higher redshifts could have diffuse stellar halos, if GCs originate from nuclei of dwarfs. Dispersions in heavier elements observed in some GCs (e.g., NGC 2419 and M22) can be closely associated with radial metallicity gradients of dwarfs, where GC merging is possible. GC merging however could be important for GC formation processes only for a minor fraction of GCs, and other physical processes such as secondary star formation from AGB ejecta (D’Ercole et al. 2008), capture of diffuse interstellar gas and molecular gas for secondary star formation (Pflamm-Altenburg & Kroupa 2008; Bekki & Mackey 2009), and gas fueling to nuclei of nucleated dwarfs (Bekki & Freeman 2003) could be also important for GCs with multiple stellar populations. We plan to extensively discuss the relative importance of these physical processes in the formation of GCs with different degrees of abundance inhomogeneity in our future papers.

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APPENDIX A: MERGING OF GC WITH DIFFERENT MASSES

We investigate whether and how GC merging can occur for models in which the two GCs have different initial masses. We here consider that (i) the models for the host dwarf of NGC 1851 and its orbital evolution are the same as those adopted the standard model, (ii) the total mass of two GCs

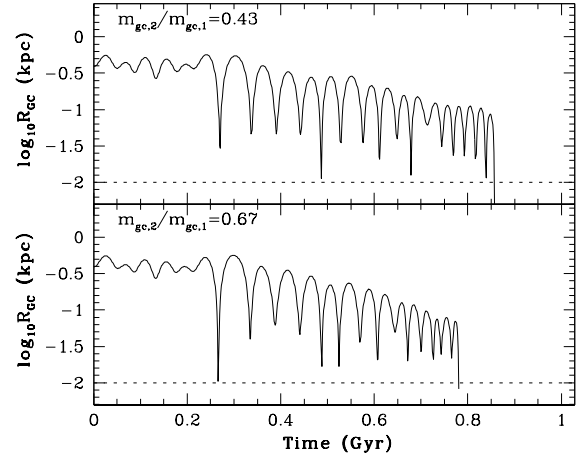


Figure A1. The same as Fig. 9 but for the GC merger models with the GC mass-ratio of 0.43 (upper, $m_{gc,1} = 7 \times 10^5 M_\odot$ and $m_{gc,2} = 3 \times 10^5 M_\odot$) and with the GC mass-ratio of 0.67 (lower, $m_{gc,1} = 6 \times 10^5 M_\odot$ and $m_{gc,2} = 4 \times 10^5 M_\odot$). Here $r_1 = r_2 = 0.2R_{s,dw}$ and the models for the dwarf and its orbit around the Galaxy are the same as those adopted in the standard model.

are fixed at $10^6 M_\odot$, and (iii) the initial masses of GC1 and GC2 ($m_{gc,1}$ and $m_{gc,2}$, respectively) are free parameters. Figure A1 shows the model in which $r_1 = r_2 = 0.2R_{s,dw}$ and $m_{gc,1}$ and $m_{gc,2}$ are $7 \times 10^5 M_\odot$ and $3 \times 10^5 M_\odot$, respectively. The adopted mass-ratio (~ 0.43) of the two GCs is consistent with the observed mass-ratio of two subpopulations in NGC 1851 by Han et al. (2009). Clearly two GCs with different masses can merge within ~ 1 Gyr in this model, and the merging timescale is quite similar to that derived for equal-mass GC models (shown in Fig. 9). Figure A1 also shows that two GCs can merge within ~ 1 Gyr for the model with $m_{gc,1}$ and $m_{gc,2}$ being $6 \times 10^5 M_\odot$ and $4 \times 10^5 M_\odot$, respectively. These results strongly suggest that GC merging is highly likely in the host dwarf of NGC 1851, if GC masses are larger than $\sim 3 \times 10^5 M_\odot$.

It should be stressed that if the mass-ratio of two GCs is less than 0.05, then the timescale of GC merging can become significantly longer than ~ 1 Gyr (for $r_1 = r_2 = 0.2R_{s,dw}$): GC merging before dwarf destruction becomes less likely for rather small GC mass-ratios. This is mainly because one of the two GCs has a small mass ($< 5 \times 10^4 M_\odot$) so that dynamical friction time scale of the GC can become rather large (i.e., can not sink rapidly into the nuclear region of the host dwarf). This result suggests that the observed mass-ratio of two subpopulations in NGC 1851 should not be so small (< 0.05), if NGC 1851 is a merger remnant of two GCs with different masses. Indeed, the observed range for the possible mass-ratio of two subpopulations (Milone et al. 2008, 2009; Han et al. 2009; Caretta et al. 2010) is consistent with the above result.